

Student ID :
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Automata Theory Course Quiz-1.a (2015-2016 Fall)
(Please use free space for draft and fit your answer to boxes.)

1. (50P) Transform NFA at the right into DFA form. ($\Sigma=\{a,b\}$)

We should start at preparing power set of Q_1 (set of states of the DFA).

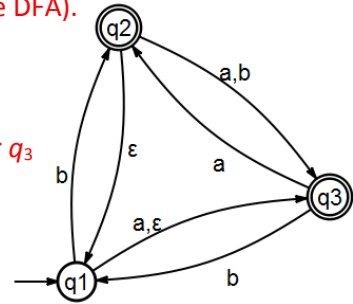
$$Q_2 = \{\emptyset, q_1, q_2, q_3, q_{12}, q_{13}, q_{23}, q_{123}\}$$

Then we can define new final states for Q_2

$$F = \{q_2, q_3, q_{12}, q_{13}, q_{23}, q_{123}\}$$
 because they includes q_2 or q_3

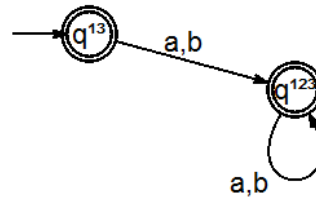
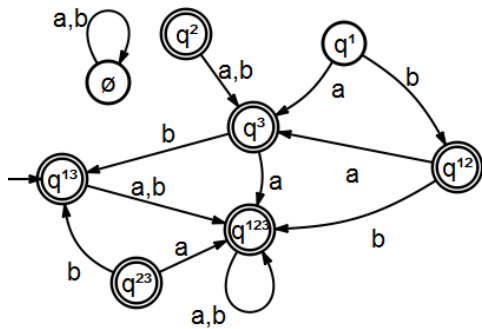
and start state

$$q = q_{13}$$
 because it can pass q_1 and q_3 by ϵ transition



Lastly, transition function

$$\begin{aligned} \delta(\emptyset, a) &= \emptyset & \delta(\emptyset, b) &= \emptyset \\ \delta(q_1, a) &= q_3 & \delta(q_1, b) &= q_{12} \\ \delta(q_2, a) &= q_3 & \delta(q_2, b) &= q_3 \\ \delta(q_3, a) &= q_{123} & \delta(q_3, b) &= q_{13} \\ \delta(q_{12}, a) &= q_3 & \delta(q_{12}, b) &= q_{123} \\ \delta(q_{13}, a) &= q_{123} & \delta(q_{13}, b) &= q_{123} \\ \delta(q_{23}, a) &= q_{123} & \delta(q_{23}, b) &= q_{13} \\ \delta(q_{123}, a) &= q_{123} & \delta(q_{123}, b) &= q_{123} \end{aligned}$$



2. (50P) When all ϵ transitions are deleted from the NFA at the top, the system transforms a DFA. For this new DFA, write the regular expression down. ($\Sigma=\{a,b\}$)

First, each state should be considered as an independent language

$$\begin{aligned} L_{q_1} &= a.L_{q_3} \cup b.L_{q_2} \\ L_{q_2} &= \epsilon \cup a.L_{q_3} \cup b.L_{q_3} \\ L_{q_3} &= \epsilon \cup a.L_{q_2} \cup b.L_{q_1} \end{aligned}$$

We should organize L_{q_1}

$$L_{q_1} = a.(\epsilon \cup a.L_{q_2} \cup b.L_{q_1}) \cup b.L_{q_2} = ab.L_{q_1} \cup (b \cup aa).L_{q_2} \cup a$$

Because L_{q_1} depend on L_{q_2} , we should reduce L_{q_2}

$$\begin{aligned} L_{q_2} &= \epsilon \cup (a \cup b).(\epsilon \cup a.L_{q_2} \cup b.L_{q_1}) = (aa \cup ba).L_{q_2} \cup (ab \cup bb).L_{q_1} \cup a \cup b \\ L_{q_2} &= (aa \cup ba)^* ((ab \cup bb).L_{q_1} \cup a \cup b) \end{aligned}$$

Now we can reorganize L_{q_1} by placing L_{q_2}

$$\begin{aligned} L_{q_1} &= (ab \cup (b \cup aa)(aa \cup ba)^*(ab \cup bb)).L_{q_1} \cup (b \cup aa)(aa \cup ba)^*(a \cup b) \cup a \\ L_{q_1} &= (ab \cup (b \cup aa)(aa \cup ba)^*(ab \cup bb))^* ((b \cup aa)(aa \cup ba)^*(a \cup b) \cup a) \end{aligned}$$