# Machine Learning

6. week

- Fuzzy Logic
- Fuzzy Inference Systems
  - Mamdani fuzzy inference
  - Sugeno fuzzy inference



## Fuzzy Logic

Fuzzy logic is an approach to computing based on "degrees of truth" rather than the usual "true or false" Boolean logic. The idea of fuzzy logic was first advanced by Lotfi Zadeh in the 1960s.

Zadeh was working on the problem of computer understanding of natural language. Natural language is not easily translated into the absolute terms of 0 and 1.

# Fuzzy Logic

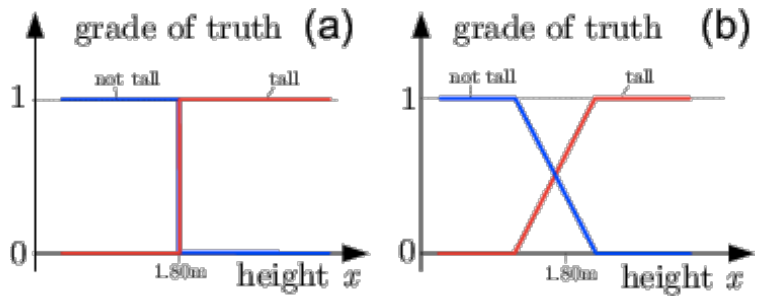
Fuzzy logic includes 0 and 1 as extreme cases of truth, but it can also use all real values between [0 1] as the membership.

For example, the result of a comparison between two things could be not "tall" or "short" but ”0.38 of tallness."



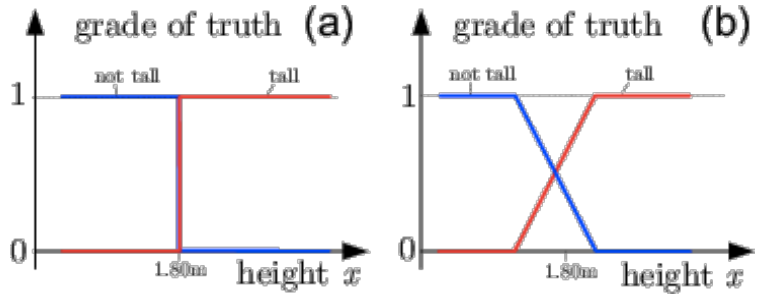
# Fuzzy Logic

According to an expert, fuzzy sets of the problem are determined with smooth words of natural language like tall, short, big, small, etc.



# Fuzzy Logic

Usually, it is expected that the sum of memberships for each x value must be 1.






# Mamdani Fuzzy Inference

The most commonly used fuzzy inference technique is the so-called Mamdani method.

In 1975, Ebrahim Mamdani built one of the first fuzzy systems to control a steam engine and boiler combination.

He applied a set of fuzzy rules supplied by experienced human operators.

## Mamdani Fuzzy Inference

The Mamdani style fuzzy inference process is performed in four steps:

- Fuzzification of the input variables
- Rule evaluation
- Aggregation of the rule outputs
- Defuzzification

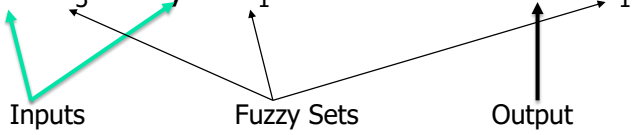




## Mamdani Fuzzy Inference

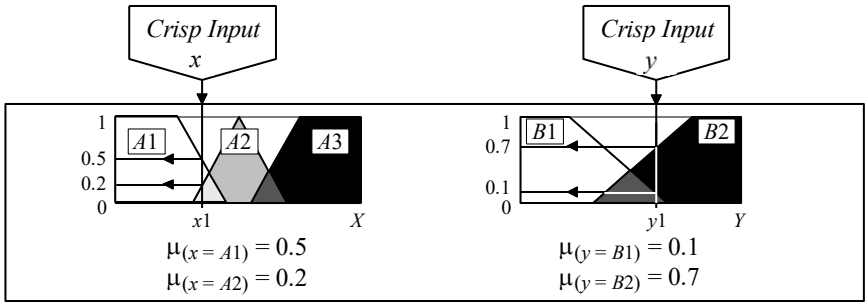
We examine a simple two-input one-output problem.

**IF** x is $A_3$ **OR** y is $B_1$ is small **THEN** z is $C_1$



# Mamdani Fuzzy Inference

**Step 1. Fuzzification:** The first step is to take the crisp inputs (x and y), and determine the membership to which these inputs belong






# Mamdani Fuzzy Inference

**Step 2. Rule Evaluation:** The second step is to take the fuzzified inputs, $m_{(x=A1)}=0.5$, $m_{(x=A2)}=0.2$, $m_{(y=B1)}=0.1$ and $m_{(y=B2)}=0.7$, and apply them to the antecedents of the fuzzy rules. If a given fuzzy rule has multiple antecedents, the fuzzy operator (AND or OR) is used to obtain a single number that represents the result of the antecedent evaluation. This number is then applied to the consequent membership function.

# Mamdani Fuzzy Inference

In order to evaluate the disjunction (union) of the rules, we use the OR fuzzy operation:

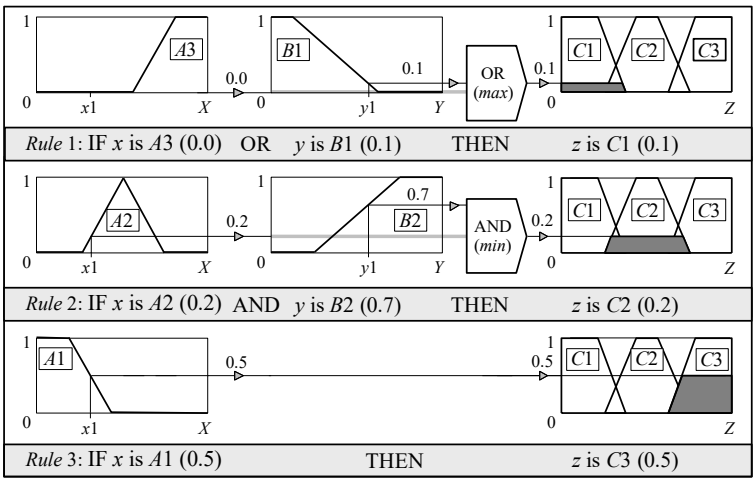
$$\mu_{A\cup B}(x) = max\,[\mu_A(x), \mu_B(x)]$$

Similarly, in order to evaluate the conjunction (intersection) of the rules, we apply the AND fuzzy operation:

$$\mu_{A\cap B}(x) = min\,[\mu_A(x), \mu_B(x)]$$



# Mamdani Fuzzy Inference



# Mamdani Fuzzy Inference

**Step 3. Aggregation of the rule outputs:**

Aggregation is the process of unification of the outputs of all rules.

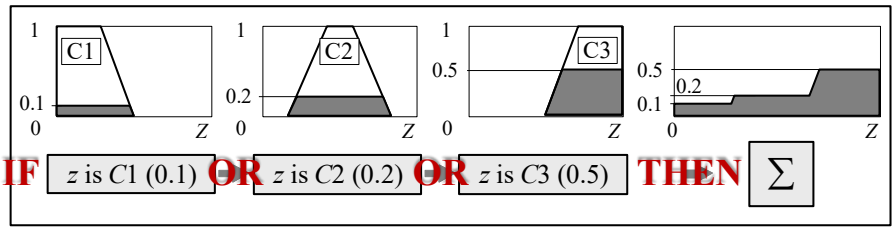
We take the membership functions of all rules and combine them into a single fuzzy set.

For summation of all rule outputs, we usually use fuzzy disjunction operation.



# Mamdani Fuzzy Inference

**Aggregation of the rule outputs**



## Mamdani Fuzzy Inference

**Step 4. Defuzzification:** The last step in the fuzzy inference process is defuzzification.

Fuzziness helps us to evaluate the rules, but the final output of a fuzzy system has to be a crisp number.

The input for the defuzzification process is the aggregate output fuzzy set, and the output is a single number.



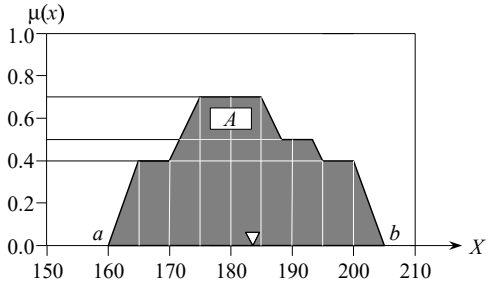
## Mamdani Fuzzy Inference

There are several defuzzification methods, but probably the most popular one is the centroid technique. It finds the point where a vertical line would slice the aggregate set into two equal masses. Mathematically this center of gravity (g) can be expressed as:

$$g=\frac{\int_a^b \mu_A(x)x dx}{\int_a^b \mu_A(x)dx}$$

# Mamdani Fuzzy Inference

Centroid defuzzification method finds the center of gravity of the fuzzy set A between [a b]. A reasonable estimate can be obtained by calculating it over a sample of points.






# Sugeno Fuzzy Inference

Mamdani-style inference is required to find the centroid of a two-dimensional shape by integrating across a continuously varying function.

In general, this process is not computationally efficient.

# Sugeno Fuzzy Inference

To avoid this problem, Michio Sugeno suggested to use a single spike (singleton) as the membership function of the rule consequent.

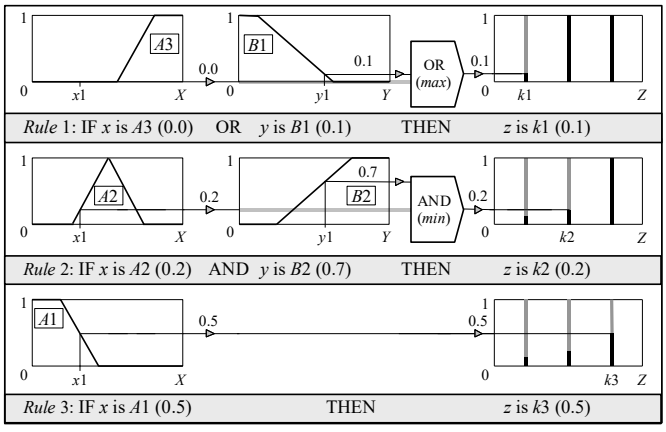
A fuzzy singleton is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.





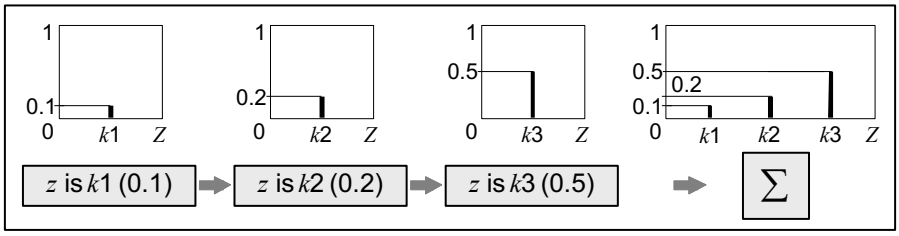
# Sugeno Fuzzy Inference

**Sugeno-style Rule Evaluation**



# Sugeno Fuzzy Inference

**Sugeno-style Aggregation of the Rule Outputs**



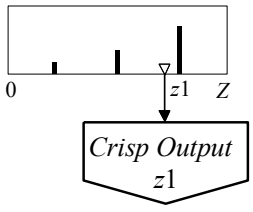



# Sugeno Fuzzy Inference

**Weighted Average (WA)**

$$WA = \frac{\mu(k1)\times k1 + \mu(k2)\times k2 + \mu(k3)\times k3}{\mu(k1)+\mu(k2)+\mu(k3)} = \frac{0.1\times 20 + 0.2\times 50 + 0.5\times 80}{0.1+0.2+0.5} = 65$$

**Sugeno-style Defuzzification**



## When using fuzzy…

1. Review model input and output variables, and if required redefine their ranges.
2. Review the fuzzy sets, and if required define additional sets on the universe of discourse. The use of wide fuzzy sets may cause the fuzzy system to perform roughly.
3. Provide sufficient overlap between neighboring sets. It is suggested that triangle-to-triangle and trapezoid-to-triangle fuzzy sets should overlap between 25% to 50% of their bases.





## When using fuzzy…

4. Review the existing rules, and if required add new rules to the rule base.
5. Adjust the rule execution weights. Most fuzzy logic tools allow control of the importance of rules by changing a weight multiplier.
6. Revise shapes of the fuzzy sets. In most cases, fuzzy systems are highly tolerant of a shape approximation.

# Presentation Task

Prepare a presentation on ANFIS (Adapted Neuro Fuzzy Inference System) with an app.