Discrete Mathematics Midterm Exam (Spring 2018)

No :

Name:

1. LOGIC (20*P*) Show that the hypotheses below lead to that conclusion: If I do not finish my program, then I will wake up refreshed.
    - If you send me an email, then I will finish my program.
    - If you do not send me an email, then I will go to sleep early.
    - If I go to sleep early, I will wake up refreshed.

a: You send me an email
b: I will finish my program
c: I will go to sleep early
d: I will wake up refreshed

$$\begin{array}{r} a \rightarrow b \\ \neg a \rightarrow c \\ \underline{c \rightarrow d} \\ \therefore \neg b \rightarrow d \end{array}$$

When we apply inference,

(1)	$\neg a \rightarrow c$	*Premise*
(2)	$c \rightarrow d$	*Premise*
(3)	$\neg a \rightarrow d$	(1), (2) *Hypothetical Syllogism*
(4)	$a \rightarrow b$	*Premise*
(5)	$\neg b \rightarrow \neg a$	(4) *Contrapositive*
(6)	$\neg b \rightarrow d$	(3), (5) *Hypothetical Syllogism*

2. RELATION (20*P*) `A={b,c,d,f}, B={a,e}` and `S={(x,y)|(x=y∈AUB)∨(x∈B∧y∈A)∨(x∈A∧y∈B)}`. By writing the set S, draw its Hasse diagram, if not possible prove it.

According to given details,

S={(a,a)(b,b),(c,c),(d,d),(e,e),(f,f),(a,b),(a,c),(a,d),(a,f),(e,b),(e,c),(e,d),(e,f),(b,a),(c,a),(d,a),(f,a),(b,e),(c,e),(d,e),(f,e)}

In order to draw Hasse diagram, we have to check three rules:

1. Reflexive: for each x∈(AUB), we can find (x,x) in S. So it's **OK**.

2. Anti-symmetric: if (x,y)∈S and (y,x)∈S, then x=y. We have (a,b) and (b,a), but a≠b. So it's **not OK**.

3. Transitive: if (x,y)∈S and (y,z)∈S, then (x,z)∈S. For (a,b) and (b,e), we cannot find (a,e) in S. So it's **not OK**.

Since 2. and 3. rules are not available, the Hasse diagram cannot be drawn here.

3. COMPLEXITY (15*P*) Find the complexity of the algorithm below for the worst case.

```
function fX(n)
    if m<=1
        return 1;
    else
        m = round((n+1)/2)
        return fX(m-1);
    end if
end function
```

We can find time function depending to "n"

n	T
1	1
2	2
4	3
8	4
16	5

We can write time function,

$T=1+\log_2 n$

and complexity for the worst case,

O(log n)

4. PIGEONHOLE (15P) In order for the number of nuts gathered by two children to be able to equal, how many nuts at least must be gathered by fifteen children? Explain how we should think about it. (Note: each child must gather at least one nut.)

If each child must gather at least 1 nut and each of 15 chilren gather different number of nuts to each other, the minimum number of nuts that can be gathered is calculated as follows:

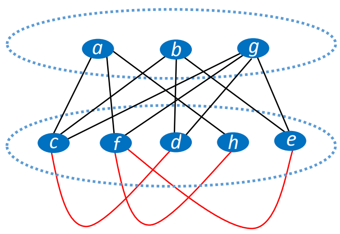
$$1 + 2 + 3 + \cdots + 15 = \frac{15 * 16}{2} = 120$$

According to this calculation, for all summation of nuts which is less than 120, we guaranteed that the numbers of nuts gathered by two children are the same.

5. GRAPH (30P) According to the adjacency matrix M of an undirected graph G, how many edges (and which ones) have to be removed from G to transform it into a complete bipartite graph? Explain your answer by showing on graph you draw.

$$\begin{array}{cc} & \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ M= & \left[\begin{array}{cccccccc} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right] \begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{array} \end{array}$$

In order to create a complete bipartiate graph, there are one efficient and economic solution. For the solution, we have to remove 3 edges from G and add 5 edges into it. When we call the vertices with the letters between 'a' and 'h' respectively,



For (a,b,g) and (c,d,e,f,h) bipartiate groups, we have to remove edges of (c-d), (e-f), (f-h)

(Note: We have to add edges of (a-d), (a-e), (b-f), (b-h), (g-h) into G. But because it is asked only the edges to remove in the question, students do not have to find the edges to add.)