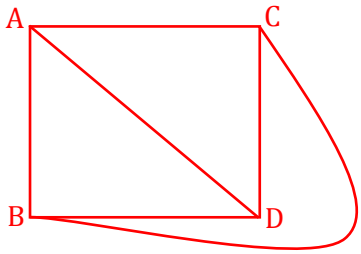
Discrete Mathematics Final Exam (Spring 2018)

No :

Name:

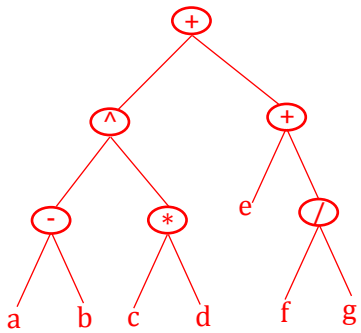
1. GRAPH (30*P*) G is a simple planar graph where it can be colored just with 4 colors and all vertices have 3 neighbors. Draw this G graph and explain your answer.



K_4

K_4 is a complete graph which can be colored with just (only) four colors. It is not only a simple (no parallel edge) planar (no edge intersection) graph, also its each vertex has 3 neighbors.

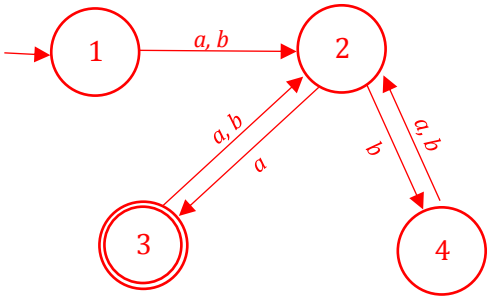
2. TREE (30*P*) Let be A=(a-b)^(c*d)+(e+f/g). By drawing A's parse tree, write its Polish (prefix) notation.



The operation in Polish notation:

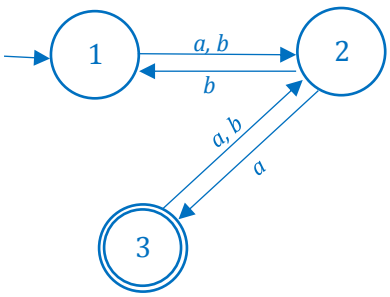
+ ^ - a b * c d + e / f g

3. AUTOMATA (30*P*) Let L be a regular language on {*a*, *b*}*. It accepts the words with even length where the last letter is "*a*". Draw deterministic finite state automaton transition diagram of L.



1: Initial state (there is no letter yet)
2: Odd-length words
3: Even-length words which finishes with "*a*"
4: Even-length words which finishes with "*b*"

OR



4. BAYES (10P) In Turkey, the incidence of diabetes is 33%. A new method which is called Diabetic Retinopathy is used to diagnose diabetes by scanning eyes of patients only. But its wrong prediction ratio in really diabetic patients is 20% and it can detect 90% of healthy people correctly. When a new patient candidate take a positive (diabetes) from this test, how can we comment his/her situation according to Bayes theorem?

D: Diabetes disease

$P(D)$=0.33 $P(\sim D)$=0.67

$P(-|D)$=0.20 $P(+|D)$=1-0.20=0.80

$P(-|\sim D)$=0.90 $P(+|\sim D)$=1-0.90=0.10

$$P(+) = P(+|D)P(D) + P(+|\sim D)P(\sim D) = 0.80 * 0.33 + 0.10 * 0.67 = 0.264 + 0.067 = 0.331$$

$$P(D|+) = \frac{P(+|D)P(D)}{\mathrm{P}(+)} = \frac{0.80 * 0.33}{0.331} \cong 0.80$$

$$P(\sim D|+) = 1 - P(D|+) \cong 0.20$$

Since $P(D|+) > P(\sim D|+)$, he/she is probably a diabetes patient.