

Discrete Mathematics Midterm 2a Exam (Spring 2017)

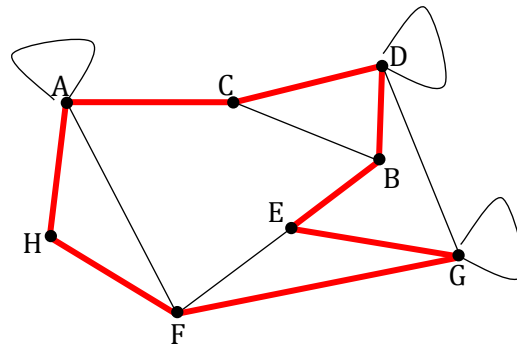
No :

Name:

1. (GRAPH) According to the given adjacency matrix M of the undirected graph,

- a. (30P) By finding a Hamilton cycle, write a gray code representing vertex neighborhoods.
- b. (20P) If the last vertex in M was not there, comment how the answer would change.

$$M = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$



Because there are 8 vertices, we need 3 bits for a gray code.

A - C - D - B - E - G - F - H

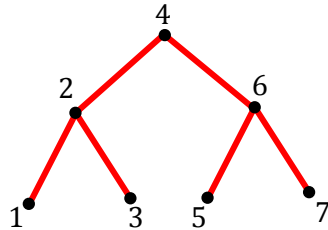
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If the last vertex was not there, we can not design a gray code. In order to design a gray code, we need always 2^n vertices.

2. (TREE) According to $X=\{1,2,3,4,5,6,7\}$

- a. (30P) How should we change order of the numbers so that the height of binary tree is two?
- b. (20P) How can we generalize the solution for any unsorted set of numbers with N elements?

Its tree must be



The element 4 must be at the first, then 2 and 6 (or 6 and 2). Lastly, it is not important the order of the left four elements (1,3,5,7).

So we can give an example as $X=\{4,2,6,1,3,5,7\}$

For any unsorted set of numbers with N elements:

1. Because we need always to find the median value, at first we should sort the set of numbers.
2. Find median value of the set of numbers.
3. Divide the set of numbers into two subsets according to the found median value.
4. For each subset of numbers, go to step 2 (find median and divide two parts)