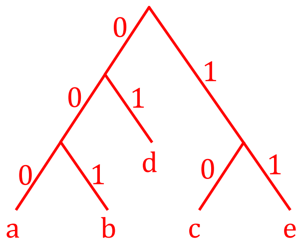
Discrete Mathematics Final Exam (Spring 2017)

No :
Name:

1. Huffman Code (25P) In a corpus where its alphabet is A = {a, b, c, d, e}, accepted words are expressed as $a^{7n}b^{8n}c^{12n}d^{21n}e^{11n}$. Here "$a^3$" means "aaa". For such a corpus, make the average bit length per letter minimum.

letter	freq
a	7
b	8
c	12
d	21
e	11



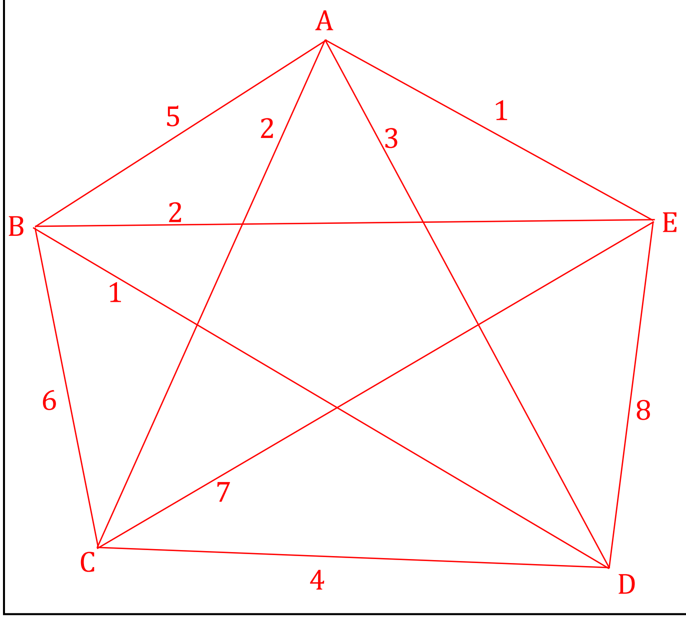
letter	bits	len
a	000	3
b	001	3
c	10	2
d	01	2
e	11	2

L= average bit length per letter

$$L = \frac{3*7+3*8+2*12+2*21+2*11}{7+8+12+21+11} = \frac{133}{59} = 2.25$$

2. Dijkstra (25P) According to the given weighted adjacency matrix (where 0 means there is no edge, otherwise there is an edge and its weight is that), show how computer can find all shortest paths from the first vertex in the matrix to every other vertex.

$$W = \begin{bmatrix} 0 & 5 & 2 & 3 & 1 \\ 5 & 0 & 6 & 1 & 2 \\ 2 & 6 & 0 & 4 & 7 \\ 3 & 1 & 4 & 0 & 8 \\ 1 & 2 & 7 & 8 & 0 \end{bmatrix}$$



iter	A	B	C	D	E
1	0	∞	∞	∞	∞
2	0	5	2	3	1
3	0	3	2	3	1
...	...	...	...	...	...

In fact, there are more iteration. But since they do not change the values, we don't need to write other rows.

3. AUTOMATA (RegEx to CFG) Let be a language in which its regular expression is a*(baa)*b.

a. (25P) Prepare context free grammar of that language.

b. (10P) What can we absolutely tell about the letters in the words accepted by that language?

Let be A is initial variable.

$A \rightarrow aA \mid B$	This rule organizes a*
$B \rightarrow baaB \mid C$	is written for (baa)*
$C \rightarrow b$	finish it with only one b

Certainly, we can say only that each word accepted by the language have to include at least one b letter. We cannot guaranteed the number of other recursive parts a* or (baa)*.

4. Hasse Diagram (15P) Write all relations hidden in the Hasse diagram below.

R = {(a,a), (a,b), (a,c), (a,d), (a,e), (a,f),
(b,b), (b,d), (b,e), (b,f),
(c,c), (c,d), (c,e), (c,f),
(d,d), (d,f),
(e,e), (e,f),
(f,f) }