Discrete Mathematics Midterm Exam (Spring 2015)

No :
Name:

1. LOGIC (20P) Prove the following logical inference problem by using natural deduction for propositional logic.

$$\begin{array}{r} p \rightarrow q \\ q \rightarrow p \\ \neg r \\ q \rightarrow r \\ \hline \therefore \quad \neg q \end{array}$$

(1)	$p \rightarrow q$	*Premise*
(2)	$q \rightarrow r$	*Premise*
(3)	$p \rightarrow r$	(1), (2), *Hypothetical Syllogism*
(4)	$\neg r$	*Premise*
(5)	$\neg p$	(3), (4), *Modus Tollens*
(6)	$q \rightarrow p$	*Premise*
(7)	$\neg q$	(5), (6), *Modus Tollens*

2. RELATION (20P) Design a Hasse diagram so that is corresponds to this set:
$S = \{(a, b) \mid a \in N, b \in N, 2 \leq a \leq 50 \leq b \leq 80, b \% a = 0\}$

When we check 3 rules (1. Reflexive, 2. Antisymmetric, 3. Transitive) for the set. For each element "a" of S, there must be a (a, a) pair. But there is only one element (50, 50) for reflexive property. But because of (50, 50), S has both transitive and antisymmetric properties. Thus we cannot draw a Hasse diagram. We can only draw Hasse diagram for a subset {(50,50)} of S. Hasse diagram of that subset must have only one point without any line.

3. COMPLEXITY (20P) For the worst case time, find the complexity of the pseudo code given below.

```
procedure f(n)
    n = n – (n % 2)

    if  n ≤ 1 then
        return 1
    else
        return (3 * f(n) - 5)
end if
```

Since in recursion step, n value of recursive function f(n) does not be decreased, procedure run forever. Therefore the complexity of the procedure is ∞.

4. INDUCTION (20*P*) Using mathematical induction, prove that for $\forall n \geq 0$.

$$(n+1).2^{n+1} = \sum_{k=0}^{n} (k+2)2^k$$

1. for n=0, the equation will be 2 = 2 (true)
2. instead of n, if we write n+1, the equation will be true because of

$$(n+2).2^{n+2} = \sum_{k=0}^{n+1} (k+2)2^k$$

$$(n+2).2^{n+2} = (n+3)2^{n+1} + \sum_{k=0}^{n} (k+2)2^k$$

$$(n+2).2^{n+2} = (n+3)2^{n+1} + [(n+1).2^{n+1}]$$

$$2(n+2) = (n+3) + (n+1)$$

$$2n+4 = 2n+4$$

5. COMBINATORICS (20*P*) Let be seven coins in a purse (or a wallet). How many possible combinations of these coins can be formed? (1KRŞ, 5KRŞ, 10KRŞ, 25KRŞ, 50KRŞ, 1TL)

We should write first equation:

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 7$$

Here each x value represent one of coin types (1, 5, 10, 25, 50, and 100).

Now we can consider this equation as a generalized permutation question:

$$\binom{7+6-1}{7} = \binom{12}{7} = \frac{12!}{7!\,5!} = 792$$