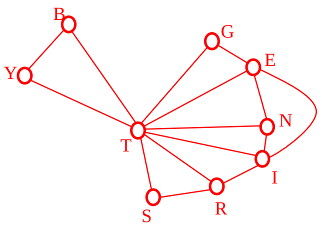
Discrete Mathematics Final Exam (Spring 2015)

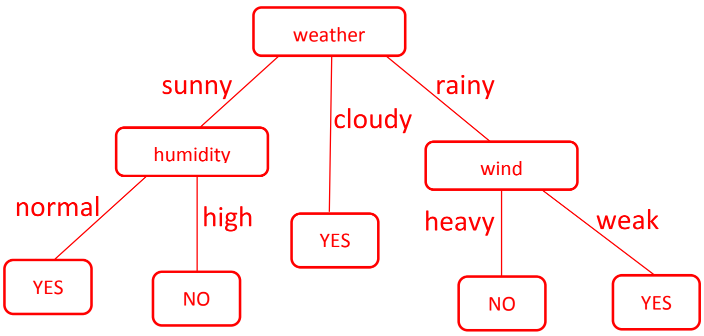
No :
Name:

1. (30P) For neighborhood of 9 countries, first draw the graph model of above map, then compute required number of colors for coloring the map. (Note that: Use these abbreviations for neighbors: Türkiye-T, Yunanistan-Y, Bulgaristan-B, Gürcistan-G, Ermenistan-E, Nahçıvan-N, İran-I, Irak-R, Suriye-S)



Assume we first paint T in red, then we cannot use red again for other 8 countries. Similarly, we can paint other countries, but here the most important neighborhood is among T, N, I, and E countries. Because all of them touch to each other. For this reason at least we need 4 colors.

2. (30P) By considering this sentence "You can play football, if the weather is cloudy, or if the weather is sunny and the humidity is normal, or if the weather is rainy and the wind is weak", draw the decision tree for the playing football problem. (Note that: the weather can be one of cloudy or sunny or rainy, the humidity can be one of normal or high, the wind can be one of weak or heavy.)



3. (30P) According to the grammar rules $A \to a\overset{1}{A}c \mid b\overset{2}{B}a$, $B \to a\overset{3}{B}a \mid b\overset{4}{B}b \mid \overset{5}{ca}$, can we derivate the word "*aabababcabababaacc*"? If so, show how. If not, explain why not.

By using five grammar rules, we can derivate the word in question as below.

$A \xrightarrow{1^*} aaAcc \xrightarrow{2} aabBacc \xrightarrow{3} aabaBaacc \xrightarrow{4} aababBbaacc \xrightarrow{3} aababaBabaacc$

$\xrightarrow{4} aabababBbabaacc \xrightarrow{5} aabababccbabaacc$

4. (10*P*) For the backgammon game, how many are there different state? (Note that: the game is played by 15 checkers for each player and 24 points on board. But student should think two more points for removed and blocked checkers.)

If we can think that checkers of different players can stand the same point on top, then the problem can be represented by a generalized permutation.

$x_1 + x_2 + x_3 + ... + x_{26} = 30$ thus we can solve it by $\binom{n+r-1}{r}$
Because of n=26 and r=30,

$$\binom{26+30-1}{30} = \binom{55}{30} = \frac{55!}{30!\,25!}$$