Discrete Mathematical Structure Midterm Exam (Spring 2013)

No :
Name:

1. LOGIC
    a. Suppose the variable x represents people and the variable y represent movies, and let $S(x;y)$ mean x saw y, $L(x;y)$ mean x like y and $A(y)$ mean y won an award. Write the following two English statements into formal logical expressions using the above predicates and any logical predicates and quantifiers.
        i. ($5P$) No one liked every movie he has seen.

$$\neg\exists x \forall y S(x;y) \rightarrow L(x;y)$$

        ii. ($5P$) Someone has never seen a movie that won an award.

$$\exists x \forall y S(x;y) \rightarrow \neg A(y)$$

    b. ($10P$) Prove the following logical inference problem by using natural deduction for propositional logic (eight rules).

$$\begin{array}{r} p \\ p \rightarrow q \\ s \vee r \\ \therefore\ \ r \rightarrow \neg q \\ \hline s \end{array}$$

(1)	p	*Premise*
(2)	$p \rightarrow q$	*Premise*
(3)	q	(1), (2), *Modus Ponens*
(4)	$r \rightarrow \neg q$	*Premise*
(5)	$\neg r$	(3), (4), *Modus Tolens*
(6)	$s \vee r$	*Premise*
(7)	s	(5), (6), *Disjunctive Syllogism*

2. RELATION
    a. ($5P$) Prove whether the following relation is an equivalence relation or not. The set Z × Z with relation R defined by

$$(m,n)\ \mathcal{R}\ (k,l) \ \Leftrightarrow \ mn > 0 \ \wedge \ kl > 0$$

1. Reflexive: for all (x,y) in ZxZ;

(xy>0) ∧ (xy>0)   but for x=0, it is false

2. Symmetric: for all (x,y) and (z,t) in Z;

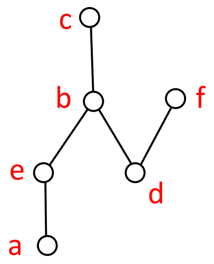
(xy>0) ∧(zt>0) ⇒ (zt>0) ∧(xy>0)

3. Transitive: for some (x,y) , (z,t) and (w,q) in Z;

((xy>0) ∧(zt>0)) ∧ ((zt>0) ∧(wq>0)) ⇒ (xy>0) ∧(wq>0)

So the relation R is not an equivalence relation.

b. (10*P*) Label this Hasse diagram so that is corresponds to this partial order:
S = {(a, a),(a, b),(a, c),(a, e),(b, b),(b, c),(c, c), . . . ,(d, b),(d, c),(d, d),(d, f),(e, b),(e, c),(e, e),(f, f)}



3. COMPLEXITY (15*P*) Find the complexity of the pseudo code below for the worst case time.

```
procedure f(n)
        n = n – (n % 2)                         (a % b  ≡  a mod b)
        if  n ≤ 1 then
                return 1
        else
                return (2 * f(n/2) + n)

end f
```

Total number of transactions depends on ($1+\log_2 n$). So for the worst case, the complexity of the procedure is *O(log n).*

4. MODULAR ARITHMETIC

a. (10*P*) Find gcd(589,247)

589 = 2*247 + 95
247 = 2*95 + 57
95 = 1*57 + 38   gcd(589,247)=19
57 = 1*38 + 19
38 = 2*19 + 0

b. (10*P*) Find $\Phi(4)^{843} \pmod{143}$

$\Phi(4)^{843} = \Phi(4)^{843 \bmod \Phi(143)} \pmod{143}$ $\Phi(4)=4*(1-0.5)=2$
$= 2^{843 \bmod \Phi(143)} = 2^{843 \bmod 120} \pmod{143}$ $\Phi(143)=12*10=120$
$= 2^3 = 8 \pmod{143}$

5. INDUCTION (10P) Using mathematical induction, prove that for $\forall n \geq 0$

$$(n+1).2^{n+1} = \sum_{k=0}^{n}(k+2)2^k$$

1. for n=0, the equation will be 2 = 2 (true)
2. for n=n+1, the equation will be true because of

$$(n+2).2^{n+2} = \sum_{k=0}^{n+1}(k+2)2^k$$

$$(n+2).2^{n+2} = (n+3)2^{n+1} + \sum_{k=0}^{n}(k+2)2^k$$

$$(n+2).2^{n+2} = (n+3)2^{n+1} + [(n+1).2^{n+1}]$$

$$2(n+2) = (n+3) + (n+1)$$

$$2n + 4 = 2n + 4$$

6. COMBINATORICS
   a. (10P) For the equation below, how many positive solutions consist of even numbers only?

$$x_1 + x_2 + x_3 + x_4 + x_5 = 18$$

The number of non-negative solutions is shown as

$$\binom{18+5-1}{18} = \binom{22}{18}$$

If we describe new even variables as $2y_i = x_i$, then the solution will be

$$\binom{9+5-1}{9} = \binom{13}{9}$$

If we also describe new non-zero and even variables as $1+z_i = y_i$, then the solution will be

$$\binom{4+5-1}{4} = \binom{8}{4} = \frac{8!}{4!\,4!} = 70$$

   b. (10P) Suppose r red balls, b blue balls, and g green balls are mixed together in a bowl. A woman selects balls at random without looking at them. There are at least 10 balls of each color in the bowl. What is the fewest number of balls she can select to be sure of having at least three balls of the same color?

According to pigeonhole principle, we must think the last selection of the woman. Before the last selection, the woman may select 2 balls for each color. So she had to already select 6 balls before the last selection in the worst case. Thus we can say that she must select 7 balls for the solution.